Midterm test for Kwantumfysica 1 - 2005-2006

Friday 10 March 2006, 9:15 - 10:00
Werkcollege-zalen group 1, 2, 3

READ THIS FIRST:

- Clearly write your name and study number on each answer sheet that you use.
- On the first answer sheet, write clearly the total number of answer sheets that you turn in.
- Note that this test has 2 questions, it continues on the backside of the paper!
- Start each question (number 1, 2) on a new answer sheet.
- The test is open book. You are also allowed to use formula sheets etc.
- If you get stuck on some part of a problem for a long time, it may be wise to skip it and try the next part of a problem first. The test is only 45 minutes.

Useful formulas and constants:

Electron mass	m_e	$= 9.1 \cdot 10^{-31}$ kg
Electron charge	$-e$	$= -1.6 \cdot 10^{-19}$ C
Planck's constant	h	$= 6.626 \cdot 10^{-34}$ Js $= 4.136 \cdot 10^{-15}$ eVs
Planck's reduced constant	$\hbar$	$= 1.055 \cdot 10^{-34}$ Js $= 6.582 \cdot 10^{-16}$ eVs

Problem T1

The position x of a particle is at some time $t = 0$ described by the normalized, real-valued wavefunction

$$\Psi(x) = Ae^{-|x/b|} \quad ,$$

with $b = 1$ nm.

a) Make a sketch of both $\Psi(x)$ and the probability density $W(x)$ for the particle's position.

b) Show that the state is normalized for $A = 1\ \text{nm}^{-1/2}$, and explain the unit of A.

c) Write down the expression for the expectation value $\langle \hat{x} \rangle$ for this state, and evaluate the answer. (If needed, you could use $\int x \cdot e^{cx} dx = \left(cx \cdot e^{cx} - e^{cx}\right)/c^2$.)

d) Explain how you could get the answer for **c)** without doing the full calculation.

e) With the particle's wavefunction as sketched, you plan to measure the position x. What is the probability for detecting a value in the range 0 nm < x < 1 nm?

f) You measure the position x, with a measurement apparatus that has a resolution of 0.1nm. You detect the particle at the position $x = 1.5$ nm. Make a sketch of the probability density $W(x)$ for the particle's position, immediately after the measurement. Explain your answer, and the width, height and area of $W(x)$ in your sketch.

Z.O.Z.

Problem T2

For this problem, write up your answers in Dirac notation.

Consider a system with a time-independent Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} ,$$

where T a kinetic-energy term and V a potential-energy term. With respect to a lowest point in the potential, defined as $V = 0$, the lowest three energy eigenstates of the system are

$$\begin{aligned} \hat{H}|\varphi_1\rangle &= E_1|\varphi_1\rangle \\ \hat{H}|\varphi_2\rangle &= E_2|\varphi_2\rangle \\ \hat{H}|\varphi_3\rangle &= E_3|\varphi_3\rangle \end{aligned} ,$$

where $E_1 < E_2 < E_3$ the three energy eigenvalues, and $|\varphi_1\rangle$, $|\varphi_2\rangle$ and $|\varphi_3\rangle$ three orthogonal, normalized energy eigenvectors. The observable $\hat{A}$, is associated with the electric dipole A of this quantum system. For this system,

$$\langle\varphi_1|\hat{A}|\varphi_1\rangle = -A_0 \quad , \qquad \langle\varphi_2|\hat{A}|\varphi_2\rangle = +A_0 \quad , \qquad \langle\varphi_3|\hat{A}|\varphi_3\rangle = +2A_0 \ ,$$
$$\langle\varphi_n|\hat{A}|\varphi_m\rangle = \langle\varphi_m|\hat{A}|\varphi_n\rangle = 4A_0 \quad , \quad \text{for all cases} \quad n \neq m.$$

Note that the states $|\varphi_1\rangle$, $|\varphi_2\rangle$ and $|\varphi_3\rangle$ are energy eigenvectors, and that they are ***not*** eigen vectors of $\hat{A}$.

a) At some time, the state of the system is (with all c_n a real-valued constant)

$$|\Psi_S\rangle = c_1|\varphi_1\rangle + c_2|\varphi_2\rangle + c_3|\varphi_3\rangle = \frac{2}{3}|\varphi_1\rangle + \frac{\sqrt{3}}{3}|\varphi_2\rangle + \frac{\sqrt{2}}{3}|\varphi_3\rangle \ .$$

Show that this is a normalized state.

b) What is for this state $|\Psi_S\rangle$ the expectation value $\langle\hat{A}\rangle$ for A, expressed in A_0?

c) At some other time, defined as $t = 0$, the state of the system is (with again all c_n a real-valued constant)

$$|\Psi_0\rangle = c_1|\varphi_1\rangle + c_2|\varphi_2\rangle + c_3|\varphi_3\rangle = \frac{2}{3}|\varphi_1\rangle + 0\,|\varphi_2\rangle + \frac{\sqrt{5}}{3}|\varphi_3\rangle \ .$$

Show that as a function of time $t > 0$, the expectation value for $\langle\hat{A}\rangle$ has oscillations at one frequency only. Determine this frequency and the oscillation amplitude (expressed in the constants that are mentioned above), for the case that the system is in $|\Psi_0\rangle$ at $t = 0$. Use the time-evolution operator (with $\hbar = h/2\pi$)

$$\hat{U} = e^{\frac{-i\hat{H}t}{\hbar}} \ .$$